angles. Although a somewhat similar arrangement occurs in $p$-aminophenol (Brown, 1951) in which distances of $2 \cdot 83,3 \cdot 13$ and $3 \cdot 18 \AA$ are found at considerably more favourable angles than those shown by hydroxylamine, it seems best to us to leave one hydrogen atom of $\mathrm{NH}_{2} \mathrm{OH}$ uninvolved in hydrogen bonding and to assume such bonds only in the 2.74 and $3.07 \AA$ distances.

It is of interest to ask what is the configuration of the molecule, and where are the hydrogen atoms. Considerable effort was made to see if our data were capable of yielding any direct information, with results described in detail elsewhere (Meyers, 1955). In summary, a detailed examination of structure factors, not given here, for three different possible arrangements of hydrogen bonds
(a) $\mathrm{O}_{1}-\mathrm{H} \cdots \mathrm{N}_{2}=2.74 \AA, \mathrm{~N}_{1}-\mathrm{H} \cdots \mathrm{O}_{3}=3.07 \AA$;
(b) $\mathrm{N}_{1}-\mathrm{H} \cdots \mathrm{O}_{2}=2.74 \AA, \mathrm{O}_{1}-\mathrm{H} \cdots \mathrm{N}_{3}=3.07 \AA$;
(c) $\mathrm{N}_{1}-\mathrm{H} \cdots \mathrm{O}_{2}=2.74 \AA, \mathrm{~N}_{1}-\mathrm{H} \cdots \mathrm{O}_{3}=3.07 \AA$
yielded, respectively, values of $R=0 \cdot 16,0 \cdot 16$ and 0.17 for all observed reflections, thus supplying no direct evidence regarding the hydrogen positions. Nevertheless, all three of these possible models lead to $\mathrm{NH}_{2} \mathrm{OH}$ molecules which, if isolated from the crystal, would have very nearly the trans configuration of symmetry $C_{s}$. This result lends some support to a model of $C_{s}$ symmetry in the gas phase, assumed by Giguère \& Liu, in so far as extrapolations of from solid to gas are valid.*

* While normally the same geometry occurs in the solid and gas, the binding forces in the solid are occasionally sufficient to produce some modifications of orientation about

It is a pleasure to acknowledge support of this research by the Office of Naval Research and to thank Dr Peter A. Howell for assistance in obtaining the diffraction patterns.

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single bonds. In hydrazine an eclipsed form was found. Presumably the singly eclipsed form of hydrazine is only a small deviation from the geometry presumed to exist in the gas phase as indicated in the first drawing of Fig. 1 in the paper by Collin \& Lipscomb (1950).

# The Structure of Guinier-Preston Zones. I. The Fourier Transform of the Diffuse Intensity Diffracted by a Guinier-Preston Zone 

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(Received 24 January 1955 and in revised form 4 March 1955)
The expression for the diffuse intensity diffracted by the Al-Cu alloy with Guinier-Preston zones is derived as a trigonometrical series with coefficients dependent on the structure. It is shown that the coefficients can be determined with the aid of the Fourier transform of the integrated diffuse intensity in relrod ( $00 l$ ). From these coefficients it is possible to deduce the features which are characteristic of the structure of Guinier-Preston zones.

## 1. Introduction

The anomalous diffraction effects in $\mathrm{Al}-\mathrm{Cu}$ and $\mathrm{Al}-\mathrm{Cu}-\mathrm{Mg}$ alloys in the age-hardened stage are well known (Preston, 1938; Guinier, 1938). They corre-
spond approximately to diffraction by a two-dimensional lattice and they are usually connected with the formation of two-dimensional copper-rich regions in the ( 001 ) planes of the matrix. The asymmetrical intensity distribution around the reciprocal lattice
points can be explained by distortions in the crystal in the neighbourhood of Guinier-Preston (G.-P.) zones due to the difference in atomic sizes of Cu and Al (Guinier, 1952). The data in the literature concerning the structure of G.-P. zones are based on calculations of scattered intensity on adequately chosen models and their comparison with the experimentally determined variation of intensity.

In this paper it is shown that the structure of G.-P. zones can be determined directly by interpretation of the Fourier transform of the diffuse intensity on the basis of a quite general model (Fig. 1).

| $\varepsilon_{3}$ |  |
| :---: | :---: |
|  | $\varepsilon_{2}$ |
|  | $\varepsilon_{1}$ |
| $\varepsilon_{\overline{1}}$ | $\Delta m_{2}$ |
|  | $\varepsilon_{\overline{2}}$ |
| $\varepsilon_{\overline{3}}$ | $\Delta m_{1}$ |

Fig. 1. Model of G.-P. zone used as a basis for the interpretation of scattered diffuse intensity.

We suppose a G.--P. zone to be formed by rectangular atomic planes ( 001 ), characterized by $\Delta m_{r} \neq 0$ and $\varepsilon_{r} \neq 0$. Here $\Delta m_{r}$ is the difference between the copper-atom fraction in the $r$ th plane of the G.-P. zone and that in the matrix; $\varepsilon_{r}$ is the displacement of the $r$ th plane of the G.-P. zone from its position in the matrix and this is measured in $\frac{1}{2} a$ as a unit of distance. The lattice of the G.-P. zone is face-centred cubic and the distribution of copper atoms within each atomic plane is assumed random. The phase relations between the individual G.-P. zones are neglected.

## 2. Intensity calculation

The amplitude of radiation diffracted by a G.-P. zone and by its neighbourhood is given by the expression (Guinier, 1952).

$$
\begin{align*}
& A=\Sigma_{(M)} f_{A} \exp (-2 \pi i \mathbf{B} \cdot \mathbf{x}) \\
& -\Sigma_{(Z)} f_{A} \exp (-2 \pi i \mathbf{B} \cdot \mathbf{x})+\sum_{(Z)} f_{B} \exp (-2 \pi i \mathbf{B} \cdot \mathbf{y}) \tag{l}
\end{align*}
$$

The first summation is taken over all atoms in the whole crystal including the G.-P. zone, the second and the third over the G.-P. zone only. $x$ and $y$ are the position vectors of atoms at the lattice points of the undistorted crystal and in the crystal with distortions respectively, $f_{A}$ is the average atomic factor of atoms outside the G.-P. zone, $f_{B}$ is the atomic factor of atoms within the G.-P. zone.

Let us begin with the calculation of the last term of the right-hand side of equation (1):

$$
\begin{align*}
\Sigma_{(Z)} f_{B} \exp & (-2 \pi i \mathbf{B} \cdot \mathbf{y}) \\
& =\sum_{-\infty}^{+\infty} \Phi_{r}(h, k) M(r) \exp \left[-\pi i l\left(r+\varepsilon_{r}\right)\right] \tag{2}
\end{align*}
$$

where $h, k, l$ are continuous variables and $\Phi_{r}(h, k)$ is the Fourier transform of the $r$ th plane in the G.-P. zone. We define the function $M(r)$ as follows:

$$
\begin{aligned}
& M(r)=\exp (-2 \pi i h), \quad r=2 n \\
& M(r)=\exp (-\pi i h), \quad r=2 n+1
\end{aligned}
$$

Then we have

$$
\begin{aligned}
& \Phi_{r}(h, k)=\left[f_{A}+\left(f_{\mathrm{Cu}}-f_{\mathrm{Al}}\right) \Delta m_{r}\right] \\
& \quad \times\{1+\exp [-\pi i(h+k)]\} \sum_{0,0}^{N_{1}-1, N_{2}-1} \exp [-2 \pi i(h u+k v)],
\end{aligned}
$$

where $f_{\mathrm{Cu}}$ and $f_{\mathrm{AI}}$ are the atomic factors of Cu and Al atoms respectively, and $N_{1}$ and $N_{2}$ are the dimensions of the G.-P. zone in the [100] and [010] directions. For simplicity we will write $\alpha_{r}=\left(f_{\mathrm{Cu}}-f_{\mathrm{AI}}\right) \Delta m_{r}$; thus

$$
\begin{align*}
& \Phi_{r}(h, k)=\left(f_{A}+\alpha_{\tau}\right)\{1+\exp [-\pi i(h+k)]\} \\
& \times \frac{\exp \left(-2 \pi i h N_{1}\right)-1}{\exp (-2 \pi i h)-1} \frac{\exp \left(-2 \pi i k N_{2}\right)-1}{\exp (-2 \pi i k)-1} \\
&=\left(f_{A}+\alpha_{r}\right) \eta(h, k) . \tag{3}
\end{align*}
$$

By introducing expression (3) into equation (2) and considering $\varepsilon_{r} \ll 1$ we obtain

$$
\begin{align*}
& F(h, k, l)=\eta(h, k)\left[\sum_{-\infty}^{+\infty} f_{A} M(r) \exp (-\pi i l r)\right. \\
& \left.\quad+\sum_{-\infty}^{+\infty} M(r)\left(\alpha_{r}-\pi i l \varepsilon_{r} f_{r}-\frac{1}{2} \pi^{2} l^{2} f_{r} \varepsilon_{r}^{2}\right) \exp (-\pi i l r)\right] \tag{4}
\end{align*}
$$

where

$$
f_{r}=f_{A}+\alpha_{r}
$$

The first term on the right-hand side of (4) obviously corresponds to diffraction by the undistorted lattice of the same size and orientation as the G.-P. zone and it has the same meaning as the second term in (1). By inserting (4) in (1) the second term of (1) and the first term of (4) cancel. The amplitude of radiation diffracted by the G.-P. zone and its neighbourhood is accordingly given by the first term on the right-hand side of (1), which corresponds to the sharp Bragg reflexion, and by the second term of (4), which appears due to the irregularities in G.-P. zones $\left(\Delta m_{r} \neq 0\right.$ and $\varepsilon_{r} \neq 0$ ). The latter term corresponds to diffuse scattering.

The diffuse intensity diffracted by the G.-P. zone is given by

$$
\begin{align*}
& \left|F_{\text {diff. }}(h, k, l)\right|^{2}=|\eta(h, k)|^{2}\left\{\pi^{2} l^{2} \sum_{r, s}\left(f_{r} \varepsilon_{r} f_{s} \varepsilon_{s}-f_{r} \alpha_{s} \varepsilon_{r}^{2}\right)\right. \\
& \quad \times M(r) M^{*}(s) \exp [-\pi i l(r-s)] \\
& \quad-\pi i l \sum_{r, s}\left(f_{r} \varepsilon_{r} \alpha_{s}-f_{s} \varepsilon_{s} \alpha_{r}\right) M(r) M^{*}(s) \exp [-\pi i l(r-s)] \\
& \left.\quad+\sum_{r, s} \alpha_{r} \alpha_{s} M(r) M^{*}(s) \exp [-\pi i l(r-s)]\right\} \tag{5}
\end{align*}
$$

For simplicity we write

$$
\begin{aligned}
A_{r s} w_{r s} & =\left(f_{r} \varepsilon_{r} f_{s} \varepsilon_{s}-f_{r} \alpha_{s} \varepsilon_{r}^{2}\right) M(r) M^{*}(s) \\
B_{r s} w_{r s} & =\left(f_{r} \varepsilon_{r} \alpha_{s}-f_{s} \varepsilon_{s} \alpha_{r}\right) M(r) M^{*}(s) \\
C_{r s} w_{r s} & =\alpha_{r} \alpha_{s} M(r) M^{*}(s) \\
w_{r s} & =M(r) M^{*}(s)=\exp \left(-\pi i \varphi_{r s}\right)
\end{aligned}
$$

On account of the relations

$$
\begin{aligned}
& A_{r s} w_{r s}=A_{s r} w_{s r}^{*} \\
& B_{r s} w_{r s}=-B_{s r} w_{s r}^{*} \\
& C_{r s} w_{r s}=C_{s r} w_{s r}^{*}
\end{aligned}
$$

equation (5) becomes

$$
\begin{align*}
& \left|F_{\text {diff. }}(h, k, l)\right|^{2} \\
& \quad=|\eta(h, k)|^{2}\left\{\pi^{2} l^{2} \sum_{r, s} A_{r s} \cos \pi\left[l(r-s)+\varphi_{r s}\right]\right. \\
& \quad-\pi l \sum_{r, s} B_{r s} \sin \pi\left[l(r-s)+\varphi_{r s}\right] \\
& \left.\quad+\sum_{r, s} C_{r s} \cos \pi\left[l(r-s)+\varphi_{r s}\right]\right\} \tag{6}
\end{align*}
$$

where

$$
|\eta(h, k)|^{2}=2[1+\cos \pi(h+k)] \frac{\sin ^{2} \pi h N_{1}}{\sin ^{2} \pi h} \frac{\sin ^{2} \pi k N_{2}}{\sin ^{2} \pi k}
$$

$\left|F_{\text {diff. }}(h, k, l)\right|^{2}$ as a function of continuous variables $h, k, l$ is not adequate for practical application. The integrated intensity of diffuse radiation in the relrod with $l$ as variable is more adequate for further analysis of the structure of G.-P. zones.

$$
I(l)_{\text {diff. }}=\iint\left|\boldsymbol{F}(h, k, l)_{\text {diff }}\right|^{2} d h d k
$$

The function $\left|F(h, k, l)_{\text {diff. }}\right|^{2}$ depends on $h$ and $k$ through the functions $|\eta(h, k)|^{2}$ and $\varphi_{r s}$. Having regard to the fact that $|\eta(h, k)|^{2}$ is appreciable only for $h$ and $k$ both not far from integers, we introduce instead of $h$ and $k$ the quantities $H+\xi_{1}, K+\xi_{2}$, where $H$ and $K$ are integers both even or odd, and $\xi_{1}$ and $\xi_{2}$ are variables $\ll 1$. The function $|\eta(h, k)|^{2}$ in this notation then becomes $\left|\eta\left(\xi_{1}, \xi_{2}\right)\right|^{2}$. The functions $\varphi_{r s}$ are
$\varphi_{r s}=\left(H+\xi_{1}\right) \times\left\{\begin{array}{l}\text { even integer ( } r, s \text { equal parity) } \\ \text { odd integer }(r, s \text { unequal parity) } .\end{array}\right.$
In the integration of (6) with respect to $\xi_{1}$ and $\xi_{2}$ the cosine and sine terms can be supposed constant and equal to their mean value in the integration range because of the rapid variation of the function $\left|\eta\left(\xi_{1}, \xi_{2}\right)\right|^{2}$.

Two separate cases are possible:
(a) $H$ and $K$ both even.-In this case $\varphi_{r s}$ are always equal to even integers. Equation (6) may then be written

$$
\begin{align*}
I(l)_{\text {diff. }}=4 N_{1} & N_{2}\left\{\pi^{2} l^{2} \sum_{-\infty}^{+\infty} A_{n} \cos \pi l n\right. \\
& \left.-\pi l \sum_{-\infty}^{+\infty} B_{n} \sin \pi l n+\sum_{-\infty}^{+\infty} C_{n} \cos \pi l n\right\} \tag{7}
\end{align*}
$$

where $n=r-s$ and the coefficients $A_{n}, B_{n}$ and $C_{n}$ are given by

$$
\begin{aligned}
A_{n} & =\sum_{-\infty}^{+\infty}\left(f_{r} \varepsilon_{r} f_{r-n} \varepsilon_{r-n}-f_{r} \alpha_{r-n} \varepsilon_{r}^{2}\right) \\
B_{n} & =\sum_{-\infty}^{+\infty}\left(f_{r} \varepsilon_{r} \alpha_{r-n}-f_{r-n} \varepsilon_{r-n} \alpha_{r}\right) \\
C_{n} & =\sum_{-\infty}^{+\infty} \alpha_{r} \alpha_{r-n}
\end{aligned}
$$

(b) $H$ and $K$ both odd.-For $r$ and $s$ of equal parity $\varphi_{r s}$ are even integers, for unequal parity $\varphi_{r s}$ are equal to odd integers. The detailed calculation leads to

$$
\begin{align*}
I(l)_{\text {diff. }} & =4 N_{1} N_{2}\left\{\pi^{2} l^{2} \sum_{-\infty}^{+\infty}(-1)^{n} A_{n} \cos \pi l n\right. \\
& \left.-\pi l \sum_{-\infty}^{+\infty}(-1)^{n} B_{n} \sin \pi l n+\sum_{-\infty}^{+\infty}(-1)^{n} C_{n} \cos \pi l n\right\} \tag{8}
\end{align*}
$$

where $A_{n}, B_{n}$ and $C_{n}$ are defined as before.

## 3. Fourier transform of integrated diffuse intensity in relrod (00l)

The intensity diffracted by G.-P. zones was expressed in the preceding section as a trigonometrical series with coefficients $A_{n}, B_{n}$ and $C_{n}$. These include $\Delta m_{r}$ and $\varepsilon_{r}$, characteristic of the structure of the G.-P. zone. In this section we will deal with the determination of $A_{n}, B_{n}$ and $C_{n}$ on the basis of analysis of the integrated diffuse intensity in relrod ( $00 l$ ) obtained experimentally.

The cosine Fourier transform of $I(00 l)_{\text {diff. }}$ is accordingly (7) given by

$$
\begin{align*}
T(x) \equiv & \int_{0}^{l_{0}} I(00 l)_{\text {diff. }} \exp \left(-\beta l^{2}\right) \cos \pi l x d l \\
& =K\left\{\pi^{2} \int_{0}^{l_{0}} l^{2} \sum_{-\infty}^{+\infty} A_{n} \exp \left(-\beta l^{2}\right) \cos \pi l n \cos \pi l x d l\right. \\
& -\pi \int_{0}^{l_{0}} l \sum_{-\infty}^{+\infty} B_{n} \exp \left(-\beta l^{2}\right) \cos \pi l x \sin \pi l n d l \\
& \left.+\int_{0}^{l_{0}} \sum_{-\infty}^{+\infty} C_{n} \exp \left(-\beta l^{2}\right) \cos \pi l n \cos \pi l x d l\right\} \tag{9}
\end{align*}
$$

where $\exp \left(-\beta l^{2}\right)$ has the meaning of the usual artificial temperature factor and $l_{0}$ is the upper limit of integrated intensity measured. $K$, here used as mean. ing the scale factor, should not be confused with the integer $K$ in the preceding section.

The coefficients $A_{n}, B_{n}$ and $C_{n}$ can be divided into one part not depending on variable $l$ and containing merely $\Delta m_{r}$ and $\varepsilon_{r}$, and a second part containing functions of $l$ alone.

Following the definition in § 1, we have

$$
f_{r}=f_{A}+\left(f_{C u}-f_{A 1}\right) \Delta m_{r} .
$$

Owing to the small concentration of copper in solid solutions we can replace $f_{A}$ by $f_{\mathrm{Al}}$ and $\Delta m_{r}$ by the
atomic fraction of copper. Using further the approximation $f_{\mathrm{Cu}}=Z_{\mathrm{Cu}} f(l), f_{\mathrm{Al}}=Z_{\mathrm{Al}} f(l)$, where $Z_{\mathrm{Cu}}$ and $Z_{\text {Al }}$ are equal to the number of electrons in the copper and aluminium atoms respectively, these approximations lead to

$$
\begin{align*}
& f_{r}=\left[Z_{\mathrm{Al}}+\left(Z_{\mathrm{Cu}}-Z_{\mathrm{Al}}\right) m_{r}\right] f(l)=Z_{r} f(l) \\
& \alpha_{r}=\left[Z_{\mathrm{Cu}}-Z_{\mathrm{Al}}\right] m_{r} f(l)=\left(Z_{r}-Z_{\mathrm{Al}}\right) f(l) \tag{10}
\end{align*}
$$

By inserting (10) in the expressions for $A_{n}, B_{n}$ and $C_{n}$ we obtain

$$
A_{n}=a_{n} f^{2}(l), \quad B_{n}=b_{n} f^{2}(l), \quad C_{n}=c_{n} f^{2}(l)
$$

where $a_{n}, b_{n}$ and $c_{n}$ are independent of the variable $l$ and are as follows:

$$
\begin{align*}
& a_{n}=\sum_{-\infty}^{+\infty}\left[\varepsilon_{r} \varepsilon_{r-n} Z_{r} Z_{r-n}-\varepsilon_{r}^{2} Z_{r}\left(Z_{r-n}-Z_{\mathrm{Al}}\right)\right] \\
& b_{n}=\sum_{-\infty}^{+\infty}\left(Z_{r}-Z_{\mathrm{Al}}\right)\left(Z_{r+n} \varepsilon_{r+n}-Z_{r-n} \varepsilon_{r-n}\right) \\
& c_{n}=  \tag{ll}\\
& =\sum_{-\infty}^{+\infty}\left(Z_{r}-Z_{\Delta \mathrm{A}}\right)\left(Z_{r-n}-Z_{\mathrm{Al}}\right)
\end{align*}
$$

The function $f(l)$ is so chosen that $Z_{C u} f(l)$ and $Z_{\text {AI }} f(l)$ represent the atomic factors at the temperature of measurement.

Equation (9) takes the form

$$
\begin{align*}
& \begin{aligned}
T(x)=K\left\{\pi^{2} \sum_{-\infty}^{+\infty} a_{n} \psi(n-x)-\right. & -\pi \sum_{-\infty}^{+\infty} b_{n} \chi(n-x) \\
& \left.+\sum_{-\infty}^{+\infty} c_{n} \gamma(n-x)\right\},
\end{aligned} \\
& \text { where } \tag{12}
\end{align*}
$$

$$
\begin{aligned}
\psi(n-x) & =\int_{0}^{l_{0}} l^{2} f^{2}(l) \exp \left(-\beta l^{2}\right) \cos \pi l(n-x) d l \\
\chi(n-x) & =\int_{0}^{l_{0}} l f^{2}(l) \exp \left(-\beta l^{2}\right) \sin \pi l(n-x) d l \\
\gamma(n-x) & =\int_{0}^{l_{0}} f^{2}(l) \exp \left(-\beta l^{2}\right) \cos \pi l(n-x) d l
\end{aligned}
$$

## 4. Interpretation of the Fourier transform

The Fourier transform of the integrated diffuse scattering according to (12) is represented as the sum of three functions $\psi, \chi, \gamma$ having non-zero values only in the immediate neighbourhood of $x=n$. We determine the coefficients $a_{n}, b_{n}$ and $c_{n}$ by the method of Flinn, Averbach \& Rudman (1954).

The function $\chi(n-x)$ has a non-zero first moment with respect to $x=n$ and a zero area. Functions $\psi(n-x)$ and $\gamma(n-x)$ have a zero first moment and a non-zero area (Fig. 2).

(a)

(b)

(c)

Fig. 2. Functions (a) $\psi$, (b) $\chi$, (c) $\gamma$ calculated without artificial temperature factor; $l_{0}=5$.

$$
\begin{align*}
& \int_{n-\delta_{1}}^{n+\delta_{1}}(n-x) T(x) d x=-K \pi b_{n} \int_{-\delta_{1}}^{\delta_{1}} \xi \chi(\xi) d \xi \\
& =-K \pi b_{n} M\left(\delta_{1}\right), \\
& \int_{n-\delta_{2}}^{n+\delta_{2}} T(x) d x=K\left[\pi^{2} a_{n} \int_{-\delta_{2}}^{\delta_{2}} \psi(\xi) d \xi+c_{n} \int_{-\delta_{2}}^{\delta_{2}} \gamma(\xi) d \xi\right]  \tag{13}\\
& =K\left[\pi^{2} a_{n} A_{1}\left(\delta_{2}\right)+c_{n} A_{2}\left(\delta_{2}\right)\right] \text {, } \\
& \int_{n-\delta_{3}}^{n+\delta_{3}} T(x) d x=K\left[\pi^{2} a_{n} \int_{-\delta_{3}}^{\delta_{3}} \psi(\xi) d \xi+c_{n} \int_{-\delta_{3}}^{\delta_{3}} \gamma(\xi) d \xi\right] \\
& =K\left[\pi^{2} a_{n} A_{1}\left(\delta_{3}\right)+c_{n} A_{2}\left(\delta_{3}\right)\right] .
\end{align*}
$$

The limits of integration $\delta_{2}, \delta_{3}$ are chosen with advantage at the first and the second zero point of the function $\psi$. This enables the coefficients $a_{n}$ and $c_{n}$ to be separated.

The equations (13) allow us in principle to determine the coefficients $a_{n}, b_{n}, c_{n}$ except for a scale factor $K$. Therefore we suppose the coefficients $\left(K a_{n}\right),\left(K b_{n}\right)$, ( $K c_{n}$ ) to be known and we look for the possibility of determining $m_{r}$ and $\varepsilon_{r}$. We begin by analysing the coefficients $c_{n}$ according to (11).

These coefficients contain $m_{r}$ only. According to the literature dealing with the structure of the G.-P. zones (Jagodzinski \& Laves, 1949; Guinier, 1952; Gerold, 1954) we suppose $m_{r}$ to be appreciable only in one or two neighbouring planes of the G.-P. zone. We distinguish, therefore, further the G.-P. zones odd and even (Fig. 3). In the case of the odd G.-P. zones


Fig. 3. (a) Odd G.-P. zone. (b) Even G.-P. zone.
we suppose that only $m_{0}$ is appreciable and that $m_{r}$ decreases rapidly as $r$ goes away from zero. In the case of even G.-P. zones $m_{0}$ and $m_{1}$ are appreciable and $m_{r}$ decreases rapidly in both directions. In the following we carry out the analysis for the odd G.-P zones only, this being more probable owing to the strong interaction of copper and aluminium atoms, as stated by Jagodzinski \& Laves (1949). That this assumption is true is shown in the Appendix on the basis of experimental data.

If the odd G.-P. zone is symmetrical, it holds $m_{r}=m_{\bar{r}}$. If asymmetrical we may introduce $\bar{m}_{r}$ by definition

$$
\bar{m}_{r}=\frac{1}{2}\left(m_{r}+m_{\bar{r}}\right) .
$$

By assuming the rapid decrease of $m_{r}$ from a maximum as $r$ goes away from zero the coefficient $c_{n}$ is given approximately for both symmetrical and asymmetrical zones by

$$
\begin{equation*}
c_{n}=\left(Z_{\mathrm{Cu}}-Z_{\Delta \mathrm{I}}\right) \sum_{-\infty}^{+\infty} \bar{m}_{r} \bar{m}_{r-n} \tag{14}
\end{equation*}
$$

The accuracy of (14) exceeds that of experimentally determined $c_{n}$. It is therefore not possible to determine from the analysis of coefficients $c_{n}$ the separate values of coefficients $m_{r}$ and $m_{\bar{r}}$; we can obtain their average values $\bar{m}_{r}$ only. For the same reason only the determination of $\bar{\varepsilon}_{r}$, the average of $\left|\varepsilon_{r}\right|$ and $\left|\varepsilon_{r}\right|$, is possible.

In the first approximation the formula (ll) gives

$$
\left.\begin{array}{l}
\left(K c_{0}\right)=K\left(Z_{\mathrm{Cu}}-Z_{\mathrm{Al}}\right)^{2} \bar{m}_{0}^{\prime 2},  \tag{15}\\
\left(K c_{1}\right)=2\left(K C_{0}\right)^{\frac{1}{2}}\left(Z_{\mathrm{Cu}}-Z_{\mathrm{Al}}\right) \bar{m}_{1}^{\prime} / K, \\
\left(K c_{2}\right)=2\left(K C_{0}\right)^{\frac{1}{2}}\left(Z_{\mathrm{Cu}}-Z_{\mathrm{Al}}\right) \bar{m}_{2}^{\prime} V K .
\end{array}\right\}
$$

The coefficients ( $K c_{n}$ ) are used owing to the impossibility of determining the coefficients $c_{n}$ on the absolute scale. For this reason also $\bar{m}_{r}$ cannot be expressed on the absolute scale. If more precise $\bar{m}_{r} / K$ is required we make use of the second approximation

$$
\begin{aligned}
& \left(K c_{0}\right)=\left(Z_{\mathrm{Cu}}-Z_{\mathrm{Al}}\right)^{2}\left(K \bar{m}_{0}^{\prime \prime 2}+2 K \bar{m}_{1}^{\prime 2}\right) \\
& \left(K c_{1}\right)=2\left(Z_{\mathrm{Cu}}-Z_{\mathrm{Al}}\right)^{2} \bar{m}_{1}^{\prime \prime} V K\left(\bar{m}_{0}^{\prime \prime} V K+\bar{m}_{2}^{\prime} V K\right) \\
& \left(K c_{2}\right)=\left(Z_{\mathrm{Cu}}-Z_{\mathrm{Al}}\right)^{2}\left(2 \bar{m}_{0}^{\prime \prime} \bar{m}_{2}^{\prime \prime} K+\bar{m}_{1}^{\prime \prime 2} K\right)
\end{aligned}
$$

The $\bar{m}_{r}^{\prime}$ and $\bar{m}_{r}^{\prime \prime}$ correspond to the first and to the second approximation respectively.

The $\bar{\varepsilon}_{n}$ can be determined from the coefficients $b_{n}$. The first approximation of (11) gives

$$
\begin{equation*}
\left(K b_{n}\right)=2 \bar{\varepsilon}_{n}^{\prime} \gamma K \cdot Z_{\mathrm{Al}}\left(Z_{\mathrm{Cu}}-Z_{\mathrm{Al}}\right) \bar{m}_{0} / K \tag{16}
\end{equation*}
$$

in the second approximation

$$
\begin{aligned}
&\left(K b_{n}\right)=2\left(Z_{\mathrm{Cu}}-Z_{\mathrm{Al}}\right) \cdot Z_{\mathrm{Al}}\left[\bar{\varepsilon}_{n}^{\prime \prime} \vee K \cdot \bar{m}_{\mathrm{0}} \vee K\right. \\
&\left.+\bar{m}_{1} \downarrow K \cdot\left(\bar{\varepsilon}_{n+1}^{\prime}+\bar{\varepsilon}_{n-1}^{\prime}\right) \bigvee K\right]
\end{aligned}
$$

The $\bar{\varepsilon}_{n}^{\prime}$ and $\bar{\varepsilon}_{n}^{\prime \prime}$ correspond to the values of $\bar{\varepsilon}_{n}$ in the first and in the second approximation. The scale factor cannot be directly determined from the experimental data.

It is a pleasure to thank Dr K. Dornberger from Deutsche Akademie der Wissenschaften zu Berlin, for valuable discussion in the initial stage of this work. I am indebted to the Research Institute of Metals for permission to publish this paper.

## APPENDIX

For an even zone we have

$$
\begin{equation*}
\sum_{-\infty}^{+\infty}(-1)^{n} c_{n}=0 \tag{17}
\end{equation*}
$$

The diffuse intensity in relpoint (110) according to (8) and (17) is

$$
I(110)_{\text {diff. }}=\Sigma(-1)^{n} c_{n}=0
$$

which is in disagreement with experiment.

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